Cellular Automata for RL and Beyond

June 12th, 2020

Justin Stevens

Outline

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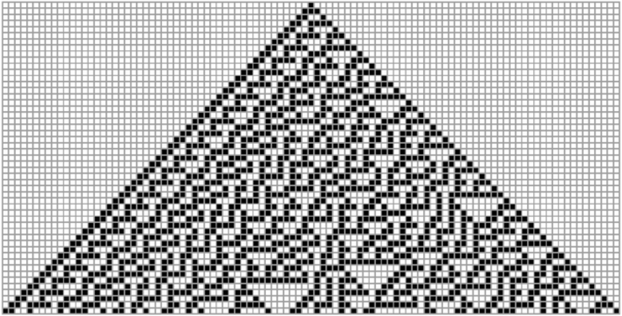
A **configuration** of the cellular automata is a function $c : \mathbb{Z}^d \to Q$, which assigns a specific value to every state. A **computation** on the cellular automata is given by the global function G specifying

$$\forall c \in Q^{\mathbb{Z}^d} \; \forall i \in \mathbb{Z}^d, G(c, i) = \delta(c(i + n_1), c(i + n_2), \cdots, c(i + n_{|N|})).$$

Example of Cellular Automata

In Wolfram's rules $d = 1$, $Q = \{0, 1\}$, $N = (-1, 0, 1)$ for the values to the left, center, and right, and $\delta : Q^3 \to Q$ is determined by the rule number. For rule 30 since $30 = 11110_2$ this is given by

Showing how the grid is evolved over time with rule 30 is shown below.



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$$p_i(t+1) = \begin{cases} p_i(t) + \alpha(1 - p_i(t)) & \text{if action i was selected} \\ (1-\alpha)p_i(t) & \text{otherwise} \end{cases}.$$

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In the case of an unfavourable response, the probability vector is updated:

$$p_i(t+1) = \begin{cases} (1-\beta)p_i(t) & \text{if action i was selected} \\ \frac{\beta}{r-1} + (1-\beta)p_i(t) & \text{otherwise} \end{cases}.$$

The parameters of the algorithm are α and β. Notice the above equations assure the probabilities sum to 1.

Learning Automata Properties

- When $\alpha = \beta$ the algorithm is called *linear reward-penalty*, L_{RP}. When $\beta = 0$ so only reward is considered, it's called *linear reward-inaction*, L_{RI}. When $a \ll b$, it's called *linear reward-* ϵ *penalty*.

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- Combines fast convergence with low computation complexity.
- Originated in mathematical psychology, but now is used in engineering.
- Also used in games with stochastic payoffs, solving NP-complete problems like graph partitioning more efficiently, data network routing, and neural network engineering.

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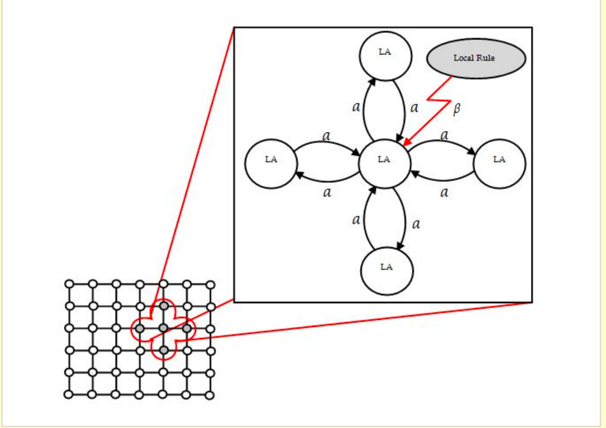
Cellular Learning Automata Intuition [2]



Assuming for the current cell, each of the neighbouring cells have m_i options for which action to select. Then the update rules is expressed as a $m_1 \times m_2 \cdots m_{|N|}$ hypermatrix.

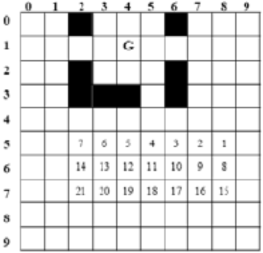
Applications of CLAs

- In [2] applications are mentioned including image processing, rumour diffusion, and modelling commerce networks.
- They also mention that if an asynchronous CLA is applied, where each cell may contain multiple LAs, but only one cell is active at a current time. This was used successfully in modelling adaptive controllers.
- In [3], the authors use CLAs to solve the channel assignment problem.
- The behaviour of CLAs can be modelled with differential equation.

Outline

Stationary Approach using Heuristics [4]

Assume we have 21 agents all trying to reach the goal with no conflicts.



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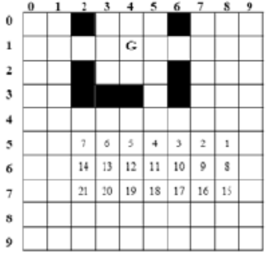

We can repeatedly apply the Bellman optimality updates to each cell.

$$
q_{i,j}^{t+1} = \begin{cases} 1 & ; \ q_{i,j}^{t} = 1 \quad \begin{pmatrix} \text{1 stands for obstacle} \\ \text{in cellular automaton} \end{pmatrix} \\[2ex] \min \left\{ \begin{array}{l} q_{i,j}^{t} \,,\; q_{i-1,j-1}^{t} + 14 \,,\; q_{i-1,j} + 10, \\ q_{i-1,j+1}^{t} + 14 \,,\; q_{i,j-1}^{t} + 10, \\ q_{i,j+1} + 10 \,,\; q_{i+1,j-1}^{t} + 14, \\ q_{i+1,j}^{t} + 10 \,,\; q_{i+1,j+1} + 14 \end{array} \right\} & \underset{\substack{r=-1,0,1 \\ s=-1,0,1}}{\forall\, q_{i+r,j+s}^{t} \neq 1} \ ;O.W. \end{cases}
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Centralized Planner

The proposed algorithm in [4] uses a centralized planner. Their basic idea is to store the current q values for each cell in a table, call it h. Then an optimal direction, dir for each cell can be computed. If multiple cells want to move to the same cell, we record this in the variable *collisions*.

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The formula for each cell c is then given by

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The agents then move in a direction so as to minimize f. If $\gamma = 0$, the performance will be equivalent to $A^\star$. The time complexity of the algorithm on an $m \times m$ grid with n agents is $O(n^3 m^4)$.

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The directions were used in a local collision avoidance model. They discretized the RL space into smaller grids to allow more agents to pass. They then created the local navigation map which can be seen on the right.

(a) (b) (c)

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The reinforcement learning and CA interact together in order to determine a solution, however, this is not a complete CA solution to the problem.

Crowd Simulation Demo [5]

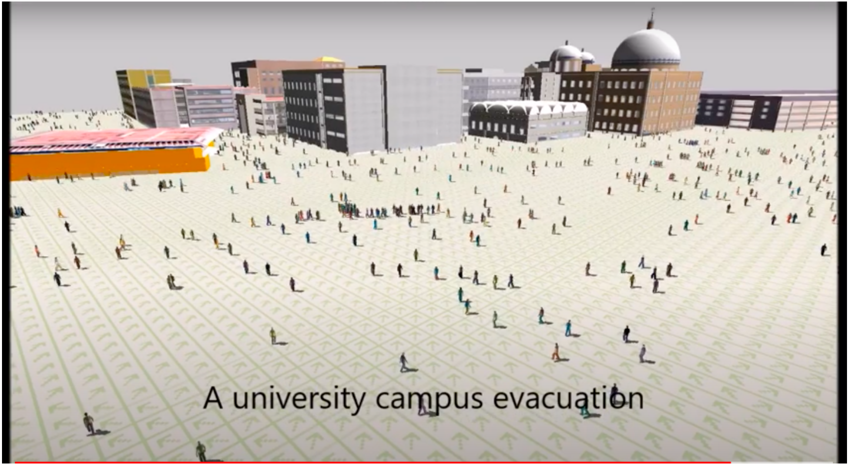

Figure 1: https://www.youtube.com/watch?v=dkx87F10x6k

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- They developed a simple model that showed some surprising results!
- Each cell is represented by a vector in $\mathbb{R}^{16}$: the first 3 values are RGB, the fourth value is a state value $0 \leq \alpha \leq 1$ that determines if the cell is alive or dead, and the other 12 are undetermined, similar to how our cells use chemical and electrical signals to grow.
- Each cell can detect the Moore neighbourhood (8-connected) around it, in addition to its own value.
- In their model they used continuous cellular automata instead of discrete so the function is differentiable. This was useful since they used a neural network to determine the updates to each cell.

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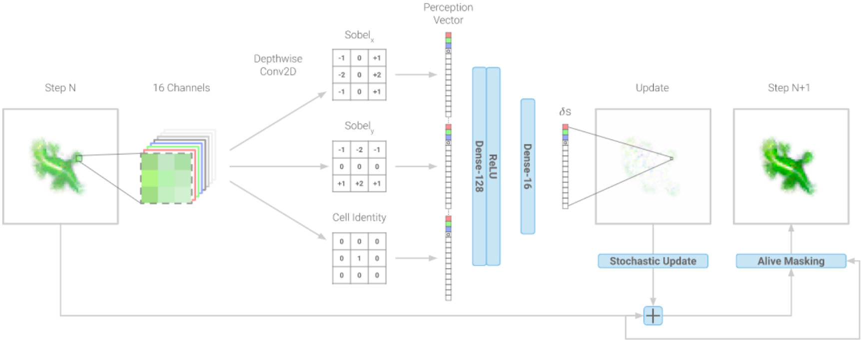
Neural Architecture [6]



A single update step of the model.

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Explanation of Model [6]

The Moore neighbourhood plus the current cell form a 3×3 table, to which three separate convolution filters are applied to each of the 16 channels. The convolutional filters are the Sobel filters, which measure the partial derivatives in the x and y direction, plus an indicator of the current cell:

$$\mathrm{Sobel}_x = \begin{pmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{pmatrix}, \mathrm{Sobel}_y = \begin{pmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{pmatrix}, \mathrm{Id} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

These are then used to create a Perception vector with $16 \times 3 = 48$ components. This vector is passed through a neural network with a Dense-128 layer followed by an RELU, and then another Dense-16 layer.

This is applied as a residual neural network, so the final δ values are added to the current cell vector in $\mathbb{R}^{16}$. There are two exceptions: we don't assume the cells have a global clock and randomly dropout some updates. Secondly, if all the cells in the Moore neighbourhood are 'dead' ($\alpha < 0.1$), then all channels are set to 0 so they don't participate in the computation.

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- Finally in the fourth experiment, they were able to generalize over rotations by multiplying the Sobel filters by the rotation matrix

$$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}.$$

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- [9] represents cellular automata as Convolutional Neural Networks.

Outline

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